Dispersion, B-integral and Kerr lens of a femtosecond pulse in a glass window, compared with published data

Fused silica and N-BK7 windows, 10 mm thick, at 800 and 1030 nm, with Gaussian and sech² pulses of 20 to 300 fs

Question

Does etendue compute correctly how a femtosecond pulse spreads in time and picks up nonlinear phase in glass? Ultrafast laser designers read the dispersion, the output duration and the B-integral of each element to decide where to compress the pulse and how much energy a lens can take. This note compares these numbers with values that follow from published material data and closed-form results.

Reference and method

A flat window is the simplest system for which each of these quantities has an exact answer. The refractive index comes from the published dispersion formulas of fused silica [1] and N-BK7 [2], and the nonlinear index from femtosecond Z-scan measurements [3]. The critical power, the Kerr phase of a Gaussian beam and the spectral broadening follow published closed-form results [4–6]. The reference values are computed from these formulas, outside etendue. The agreement criteria were set before the first measurement.

1 The system

PARAMETER	VALUE	PARAMETER	VALUE
Window	10 mm, flat faces, 25.4 mm diameter	Beam	Gaussian, 1.0 mm radius at 1/e², waist on the entrance face
Glasses	fused silica [1], N-BK7 [2]	Pulses	Gaussian 20–300 fs, sech² 300 fs, transform-limited
Wavelengths	800 and 1030 nm	Energies	1 pJ (linear), 51–701 µJ (nonlinear)

Table 1. The window and the source, as entered in etendue in laser mode. Air follows the window up to the image plane, 10 mm away.

2 Deviation of each quantity from its reference

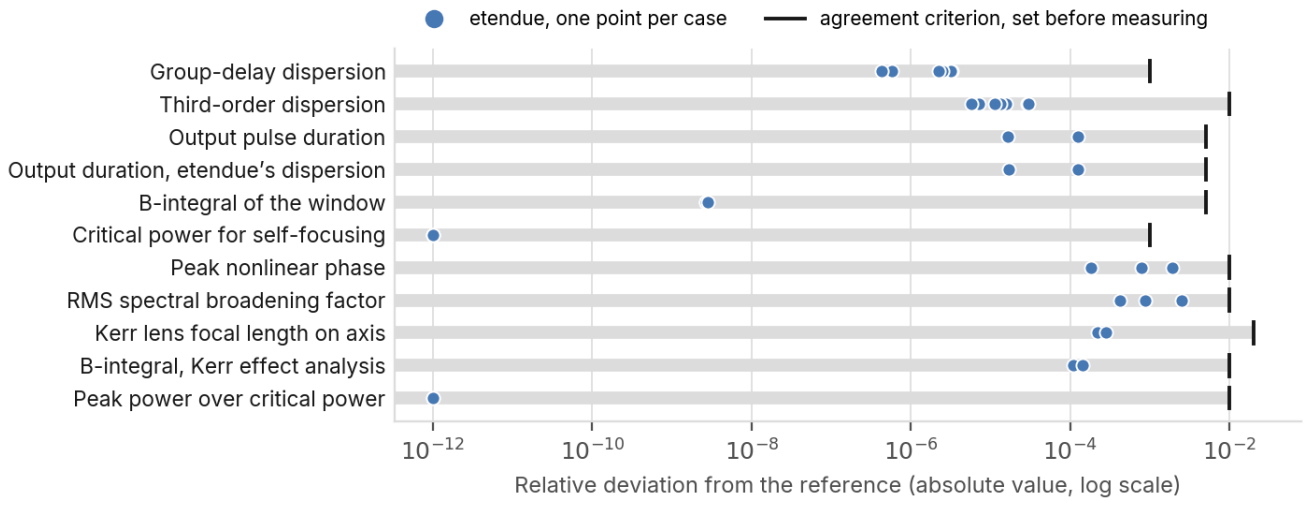


Figure 1. Relative deviation of etendue from the reference, one point per case, against the agreement criterion of each quantity (black tick). Points at 10^{-12} or below agree to machine precision. Cases and values: Table 2.

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Quantities within the agreement criterion written before the measurement.

+3.2×10⁻⁶
Largest relative deviation of the group-delay dispersion, both glasses, both wavelengths.

0.25 %
Largest deviation of all: spectral broadening by self-phase modulation at B = 1.9 rad, for a criterion of 1 %.

0.03 %
Kerr lens focal length of the window, against the parabolic Kerr phase of a Gaussian beam.

Results against the published references

38 of 38 quantities and 8 of 8 set-up checks within their criteria

Table 2. Deviation by quantity, all cases

QUANTITY	VALUES	LARGEST DEVIATION	CRITERION	VERDICT
Group-delay dispersion	8	$+3.2\times10^{-6}$	0.1 %	met
Third-order dispersion	8	$+3.0\times10^{-5}$	1 %	met
Output pulse duration	2	$+1.3\times10^{-4}$	0.5 %	met
Output duration, etendue's dispersion	2	$+1.3\times10^{-4}$	0.5 %	met
B-integral of the window	3	-2.8×10^{-9}	0.5 %	met
Critical power for self-focusing	3	$< 10^{-14}$	0.1 %	met
Peak nonlinear phase	3	-1.9×10^{-3}	1 %	met
RMS spectral broadening factor	3	-2.5×10^{-3}	1 %	met
Kerr lens focal length on axis	2	-2.8×10^{-4}	2 %	met
B-integral, Kerr effect analysis	2	$+1.4\times10^{-4}$	1 %	met
Peak power over critical power	2	$< 10^{-14}$	1 %	met

Deviation = (etendue − reference) / reference, largest over the cases. Cases: dispersion for each glass at 800 and 1030 nm (1 pJ); duration of 20 and 50 fs pulses after the silica window at 800 nm, third-order dispersion compensated at the input; B-integral and critical power at B = 0.5 rad in three windows; self-phase modulation of a 300 fs pulse at B = 0.5, 1.0 and 1.9 rad; Kerr lens for two of these windows. Values: one per case and per path (material and traced dispersion count twice).

Table 3. Dispersion of the 10 mm window

10 mm window	GDD, ref. (fs^2)	Material (fs^2)	Traced (fs^2)	TOD, ref. (fs^3)	Material (fs^3)	Traced (fs^3)
silica, 800 nm	361.620	361.621	361.620	274.97	274.98	274.97
silica, 1030 nm	189.726	189.726	189.725	411.47	411.48	411.48
N-BK7, 800 nm	446.518	446.519	446.518	321.01	321.02	321.02
N-BK7, 1030 nm	251.223	251.224	251.222	455.22	455.23	455.23

etendue gives each value twice: from the derivatives of the material index, and from the phase of the traced ray at five frequencies.

References. [1] I. H. Malitson, J. Opt. Soc. Am. 55, 1205 (1965), dispersion formula in the abstract. [2] SCHOTT, optical glass data sheets (2017-01-20), N-BK7, constants of dispersion. [3] S. R. Flom et al., Appl. Opt. 54, F123 (2015), Table 1. [4] G. Fibich, A. L. Gaeta, Opt. Lett. 25, 335 (2000), Table 1, $P_{cr} = 1.8962\ \lambda^2/(4\pi\ n_0\ n_2)$. [5] M. Sheik-Bahae et al., IEEE J. Quantum Electron. 26, 760 (1990), Eqs. (5)–(6): on-axis Kerr phase $k\ n_2\ I_0\ L$ of a Gaussian beam, whose parabolic expansion gives the Kerr lens. [6] S. C. Pinault, M. J. Potasek, J. Opt. Soc. Am. B 2, 1318 (1985): RMS spectral width after self-phase modulation, $(1 + 4B^2/3\sqrt{3})^{1/2}$, as quoted by C. Finot et al., J. Opt. Soc. Am. B 35, 3143 (2018), Sec. 3, and re-derived here by numerical integration. [7] P. E. Ciddor, Appl. Opt. 35, 1566 (1996), dry air. The broadening of a Gaussian pulse by group-delay dispersion and the dispersion orders β_2, β_3 are exact derivatives of the index, computed with 40-digit arithmetic.

Verdict

All 38 quantities meet the criteria written before the measurement, and so do the 8 set-up checks (nonlinear index read by etendue, peak power of the pulse). The margin to the criterion ranges from a factor of 4, for the spectral broadening at B = 1.9 rad, to six orders of magnitude for the B-integral.

What the agreement does not show

The critical power and the ratio of peak power to critical power agree to machine precision because etendue applies the same formula as [4]. This checks the index, the nonlinear index and the wavelength that go into it, not the physics of self-focusing.

The nonlinear index is the femtosecond value of [3] at 1030 nm. At 800 nm etendue interpolates between the 772 and 1030 nm values of [3] (2.09×10^{-20} m^2/W for fused silica). The Kerr quantities at 800 nm check the formulas, not that value.

The pulse also crosses 10 mm of air before the image plane. Both etendue and the reference count it as dry air at 20 °C (0.21 fs^2 at 800 nm; reference from [7]). The 20 fs pulse then deviates by $+1.7\times10^{-5}$.

Limits of validity

The references assume a beam that does not self-focus. With a 50 µm waist at 0.2 times the critical power, the Kerr effect analysis follows the narrowing beam and gives a B-integral 1.9 % higher and a Kerr focal length 3.0 % shorter than the linear reference. The pulse analysis keeps the linear beam and warns that its B-integral is low by about 3.3 %. No published value was available to judge this case.

Not compared, for lack of a closed-form reference: the output duration with third-order dispersion, a dispersed $sech^2$ pulse, and the self-focusing distance.