Total integrated scatter of a rough mirror, and Lambertian and ABg diffusers

The etendue non-sequential ray tracer compared with the Davies and Bennett-Porteus scatter law, the Lambertian view factors and the integral of the ABg model

The question

Does etendue scatter the right fraction of light, in the right directions? Stray-light and illumination studies rest on it. The roughness of a polished mirror sets how much light leaves the specular beam, and the scatter law of a diffusing surface sets where that light goes.

The reference

For a moderately rough surface, the fraction scattered is $1 - \exp(-(4\pi\sigma \cos\theta/\lambda)^2)$, after Davies and Bennett and Porteus, as written by Harvey et al. (2012, eq. 3). A Lambertian surface sends the fraction $\sin^2\theta$ of its light within an angle θ (Howell, cases B-12 and C-41). For the ABg model with g = 2, the fraction within $\sin\theta = u$ is $\ln(1 + u^2/B)/\ln(1 + 1/B)$, the integral given by Freniere (2012).

1 The systems

A thin collimated beam strikes a flat mirror of known roughness, and the power left in the specular beam gives the scatter. Then the same beam strikes a plate that reflects 80 % of the light diffusely, and a large detector records where the scattered light lands. Each diffuser is traced once with 200 000 rays; its noise is computed from the spread of the ray weights.

PART	VALUE
Rough mirror	flat, σ = 1 to 40 nm, 1 000 rays, 400 / 550 / 1 064 nm, 0° / 30° / 60°
Diffusing plate	reflects 80 % of the light, Lambertian or ABg
Beam on the plate	collimated, 0.1 mm radius, normal incidence, 550 nm
Rays per diffuser	200 000
Detector	plane at 20 mm (Lambertian) or 100 mm (ABg)

2 How far etendue is from the exact values

How close is the scatter? On every mirror, from 1 to 40 nm of roughness, etendue follows the exact law and not its first-order form, and the diffusers send their light where the exact laws do, within the random noise.

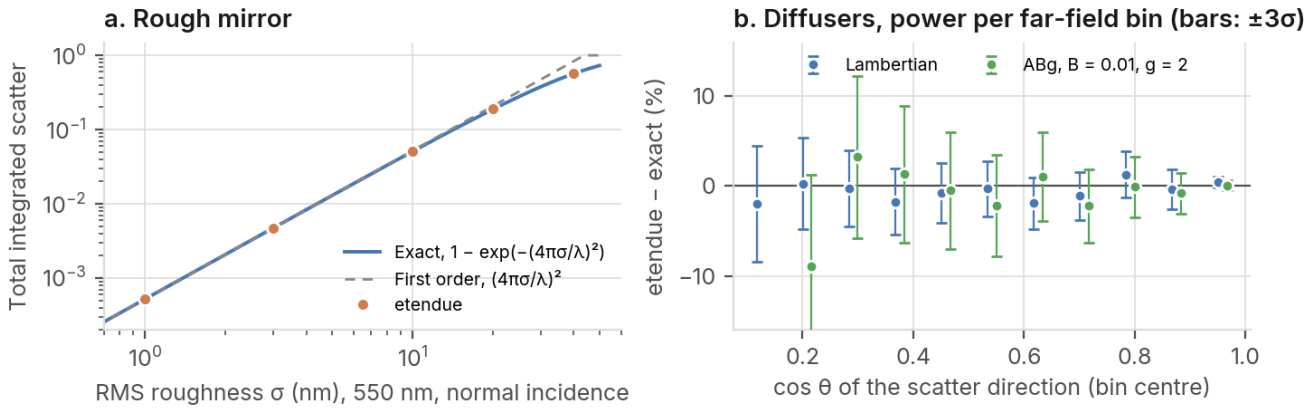


Figure 1. a. Fraction of light scattered by a flat mirror at 550 nm and normal incidence, against its RMS roughness. b. Relative difference between etendue and the exact power in each far-field bin of equal solid angle, for the two diffusers. Only the bins that the detector sees entirely are shown.

0.00 %
Largest error on the scatter of a rough mirror (σ = 1 to 40 nm), over three wavelengths and three angles.

2.6 σ
Largest difference from the exact diffuser values, over 34 Monte Carlo comparisons. All lie within 3σ.

0.7970
Fraction of the source power scattered by the Lambertian plate. Exact: 0.8.

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Comparisons within the limits written before the measurement.

References: Harvey et al., Opt. Eng. 51 (2012); Howell, Configuration Factors; Freniere (2012) · etendue, November 2026

The numbers, and what they do not say

Every comparison, the noise it was judged against, and the domain of the scatter law.

Table 2. Scatter of a flat mirror

σ (nm)	λ (nm)	θ (°)	Exact	etendue	Diff. (%)	
1	550	0	0.0005219	0.0005219	−0.00	met
3	550	0	0.004687	0.004687	−0.00	met
3	550	30	0.003517	0.003517	−0.00	met
3	550	60	0.001174	0.001174	−0.00	met
3	400	0	0.008843	0.008843	+0.00	met
3	1064	0	0.001255	0.001255	−0.00	met
10	550	0	0.05086	0.05086	+0.00	met
20	550	0	0.1885	0.1885	−0.00	met
40	550	0	0.5662	0.5662	+0.00	met

σ: RMS roughness; θ: angle of incidence. Limit: 1 % of the exact value, a half-percent error on the roughness one would infer from it.

Table 3. Plate reflecting 80 %, fractions of the source power

Quantity	Exact	etendue	Diff. (%)	Diff. / σ	
Lambertian, r ≤ 5 mm	0.0471	0.0472	+0.36	+0.8	met
Lambertian, r ≤ 10 mm	0.1600	0.1605	+0.31	+1.2	met
Lambertian, r ≤ 20 mm	0.4000	0.4006	+0.16	+0.5	met
Lambertian, r ≤ 40 mm	0.6400	0.6385	−0.23	−1.0	met
Lambertian, r ≤ 100 mm	0.7692	0.7664	−0.37	−1.9	met
ABg, r ≤ 2.5 mm	0.0105	0.0108	+3.08	+1.2	met
ABg, r ≤ 5 mm	0.0386	0.0387	+0.20	+0.2	met
ABg, r ≤ 10 mm	0.1193	0.1189	−0.31	−0.5	met
ABg, r ≤ 25 mm	0.3344	0.3345	+0.03	+0.1	met
ABg, r ≤ 50 mm	0.5277	0.5280	+0.04	+0.3	met
ABg, r ≤ 100 mm	0.6816	0.6812	−0.05	−0.4	met
Lambertian, bins (11)			−2.0 to +1.3	−2.0 to +2.2	met
ABg, bins (10)			−9.0 to +3.2	−2.6 to +1.1	met
Lambertian, total	0.8000	0.7970	−0.37	−1.3	met
ABg, total	0.8000	0.7982	−0.22	−1.0	met

r ≤ x: power landing within a radius x of the axis on the detector. A far-field bin spans 1/12 in cos θ. σ: Monte Carlo noise, from the spread of the ray weights; a value is met when it lies within 3σ.

The domain of the scatter law

etendue computes the fraction scattered by a rough mirror as $1 - \exp(-(4\pi\sigma \cos\theta/\lambda)^2)$, equation 3 of the reference, which its authors give for moderately rough surfaces and hold accurate up to a scatter of about 0.9. Its first-order form, $(4\pi\sigma \cos\theta/\lambda)^2$, is good to 1 % only up to about λ/89 of roughness, and would read 48 % high at 40 nm. The roughness entered is taken as the band-limited roughness relevant to scatter, the one the reference asks for; a total roughness measured over a wider band would overstate the scatter.

What this does not cover

The shape of the Harvey-Shack lobe, whose constants are internal, and the total scatter implied by the A coefficient of the ABg model: etendue takes the scattered fraction as an input instead. Nor polarisation or diffraction of the scattered light.

How it was computed

Each scene is built and traced as the non-sequential editor does it, with the options of the stray light analysis, and read by the stray light and encircled energy analyses of etendue. No value was tuned. As a check of the power balance, the Lambertian plate absorbs 0.203 W of the 1 W it receives, for 0.2 W expected, and the balance closes to 3.5×10^{-16} W.

References

[1] J. E. Harvey, S. Schröder, N. Choi, A. Duparré, “Total integrated scatter from surfaces with arbitrary roughness, correlation widths, and incident angles”, *Opt. Eng.* 51(1), 013402 (2012), eqs. (1)-(4), after H. Davies, *Proc. IEE* 101 (1954), and H. E. Bennett, J. O. Porteus, *J. Opt. Soc. Am.* 51 (1961). [2] J. R. Howell, *A Catalog of Radiation Heat Transfer Configuration Factors*, 3rd ed., online, cases B-12 and C-41. [3] E. R. Freniere, “Insights into the ABg BSDF model”, technical note (2012), total scatter integral for g = 2.