Thermal lens of a laser-heated fused-silica element, checked against analytical solutions

1000 W Gaussian beam at 1064 nm, steady state, heat absorbed in the bulk or in a coating

The question

A laser beam that crosses a window or a lens leaves part of its power in the glass. The glass warms and becomes a weak lens that moves the focus of the beam. This note checks the temperature and the thermal focal length that etendue computes against two published analytical models, for heat absorbed in the bulk or in a coating. The focal length agrees within 0.27 %.

The references

Hello and Vinet solved the heat equation in a cylinder heated by a Gaussian beam, with linear heat losses on every face, as a series of Bessel functions [1]. They give the temperature field for heat absorbed in the bulk and in a coating, and the optical path added by dn/dT. Chénais et al. give the focal length of a Gaussian heat deposit between insulated faces [2]. The cases keep the approximations of both below 10^{-4}.

1 The elements and the beam

Table 1. Eleven heated elements, all in fused silica (conductivity 1.38 W/(m·K), the value used in [1]), each with flat faces. The beam is a Gaussian of radius w at $1/e^2$. The bulk absorbs 50 or 100 ppm/cm; a coating absorbs a fixed share of the beam. Only the index change is counted, as in both references.

CASE	ELEMENT	BEAM RADIUS W	HEAT LOSSES	HEAT SOURCE	REFERENCE
A1 to A5	Ø6 to Ø20 × 2 mm	0.5 to 2 mm	faces insulated, edge at ambient	bulk	[2]
B1	Ø600 × 200 mm	20 mm	h = 4.90 W/(m²·K), all faces	bulk	[1] eq. (26), (29)
B2	Ø50 × 10 mm	5 mm	h = 10 W/(m²·K), all faces	bulk	[1] eq. (26), (29)
C1	Ø600 × 200 mm	20 mm	as B1	coating, 0.1 %	[1] eq. (8), (29)
C2	Ø50 × 10 mm	5 mm	as B2	coating, 0.01 %	[1] eq. (8), (29)
D1, D2	Ø4, Ø6 × 2 mm	2 mm	as A	bulk, beam clipped	Gaussian, [1] eq. (9)

2 Agreement with the references

Does etendue reproduce the published temperature and focal length? Yes, at the default mesh of the analysis. The small difference that remains comes from the mesh: twice as fine in each direction, it falls below 0.04 %.

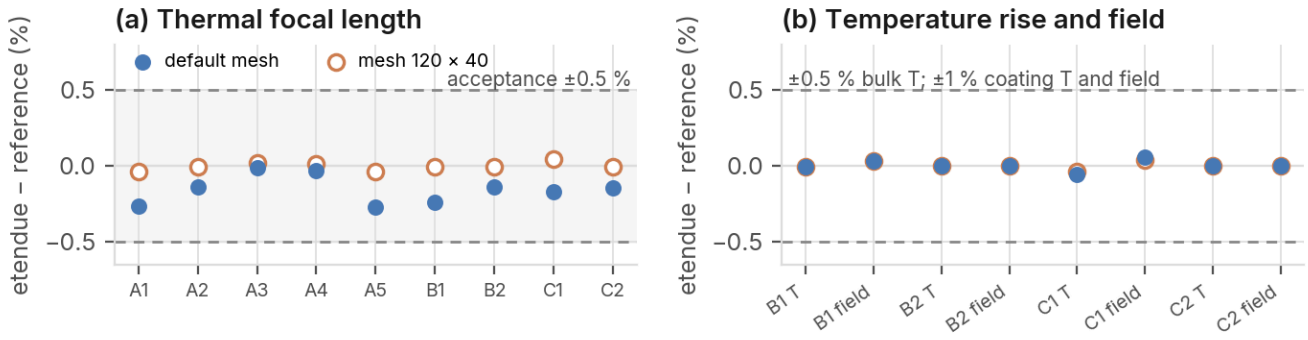


Figure 1. Difference to the reference of (a) the thermal focal length and (b) the temperature rise: at the centre (bulk heating) or the hot spot (coating), and the largest difference over the field, as a share of the peak rise. Filled: default mesh. Open: twice as fine in each direction.

19/19
Comparisons within the criterion written before the measurement.

0.27 %
Largest focal-length difference at the default mesh, 0.04 % on a finer mesh.

0.06 %
Largest temperature difference, at the centre, at the hot spot or anywhere in the element.

$3.1 \cdot 10^{-8}$
Relative difference of the power clipped by the aperture of the element.

What was compared, and where it holds

Temperature, field and focal length for bulk and coating heating; power lost at the aperture.

Table 2. Focal length and temperature rise, default mesh

CASE	ABSORBED (W)	FOCAL LENGTH, REF. (M)	FOCAL LENGTH, ETENDUE (M)	DIFF. (%)	RISE, REF. (K)	RISE, ETENDUE (K)	DIFF. (%)
A1	0.02	5.5	5.5	−0.27	—	—	—
A2	0.02	22.1	22.1	−0.14	—	—	—
A3	0.02	88.5	88.5	−0.01	—	—	—
A4	0.02	22.1	22.1	−0.03	—	—	—
A5	0.02	22.1	22.1	−0.27	—	—	—
B1	1	-201.4	-200.9	−0.24	1.770	1.770	−0.0056
B2	0.1	113.8	113.7	−0.14	3.690	3.690	+0.0001
C1	1	-207.9	-207.6	−0.17	13.070	13.062	−0.0590
C2	0.1	114.9	114.7	−0.15	6.830	6.830	−0.0001

B1 and C1 use the dn/dT of [1], which is negative, hence a diverging thermal lens. The temperature rise is taken at the centre (B), at the hot spot on the coated face (C).

Table 3. Power clipped by the aperture

CASE	APERTURE RADIUS / W	CLIPPED, REF. (W)	CLIPPED, ETENDUE (W)	REL. DIFF.
D1	1	135.3353	135.3353	$1.4{\cdot}10^{-8}$
D2	1.5	11.1090	11.1090	$3.1{\cdot}10^{-8}$

For a Gaussian beam the power outside a disc of radius a is $P{\cdot}\exp(-2a^2/w^2)$.

Table 4. Focal-length difference against the mesh (%)

MESH	B1, BULK	C1, COATING	C2, COATING
60 × 20	−0.24	−0.17	−0.15
60 × 40	−0.24	−0.20	−0.14
60 × 80	−0.24	−0.20	−0.14
60 × 100	−0.24	−0.20	−0.14
120 × 20	−0.00	+0.07	−0.01
240 × 20	+0.05	+0.13	+0.02
400 × 100	+0.05	+0.10	+0.03

Radial × axial intervals. 60 × 20 is the default of the analysis; the finest mesh it accepts is 400 × 100.

How etendue computes it

The analysis finds the power each element absorbs, in the bulk and in its coatings, solves the steady heat equation of the element in radius and depth by finite elements, and traces the beam through the resulting index field. The focal length is read from the curvature of the optical path on the axis, the quantity both references give. The beam radius in the element is updated from the heated state until it no longer changes; here one or two passes suffice.

Domain of validity

Steady state, symmetric about the axis, with a constant dn/dT and linear heat losses. The references count only the index change: the stress and bulging terms that etendue also computes were not compared, for lack of published values. Nor was the default heat loss (convection and radiation): the cases set the losses of each reference. Heated mirrors are not covered.

Mesh

The default mesh has 60 radial and 20 axial intervals, with layers tightened towards a heated coating. For the three cases of Table 4, on every mesh, the focal length stays within 0.24 % of the reference. Two effects remain. The beam crosses the lens it creates, which the references leave out: it moves the focal length of the two 200 mm parts by up to 0.07 %. Without it, the finest mesh leaves up to 0.05 % where the field matches to $1.4{\cdot}10^{-7}$: the floor of the focal-length reading.

Conclusions

For heat absorbed in the bulk or in a coating, etendue reproduces the published steady-state temperature and the thermal focal length of a laser-heated fused-silica element within 0.27 % at its default mesh, across element sizes from a few millimetres to 60 cm and focal lengths from 5 m to 200 m. The remaining difference comes from the mesh.

References. [1] P. Hello, J.-Y. Vinet, Analytical models of thermal aberrations in massive mirrors heated by high power laser beams, J. Phys. France 51, 1267-1282 (1990), eq. (6)-(10), (26), (29). [2] S. Chénais, F. Druon, S. Forget, F. Balembois, P. Georges, On thermal effects in solid-state lasers: the case of ytterbium-doped materials, Progress in Quantum Electronics 30, 89-153 (2006), eq. (III.16)-(III.18) and p. 51.