Path difference and fringe count in a Michelson interferometer

The etendue non-sequential ray tracer compared with the worked examples of OpenStax University Physics, volume 3, section 3.6

The question

Does etendue compute the optical path difference between the two arms of an interferometer correctly, to a small fraction of a fringe? A ray tracer that gets the path length wrong by a few nanometres would count fringes wrongly, and every interferometric result built on it would inherit the error.

The reference

The textbook gives two worked results. Moving one mirror by 0.315 µm at 630 nm shifts the pattern by one fringe. A 2 cm cell filled with gas in one arm shifts it by 122 fringes at 632.8 nm, which gives a refractive index of 1.0019. Both follow from the path difference $2d_1 - 2d_2$ between the arms.

1 The system

Table 1. The interferometer as the textbook describes it. A collimated beam is split by a cube into two arms, each closed by a flat mirror, and recombined on the detector. The cube plays the part of the compensating plate: each beam crosses it twice, so both arms see the same thickness of glass. Mirror 2 is then moved along its arm, or a gas cell is placed in arm 2.

PART	VALUE
Source	collimated, 5 mm radius, 100 rays, 630 nm
Beam splitter	25.4 mm cube, N-BK7, 50/50
Arms	100 mm from the splitter to each mirror
Gas cell	20 mm long in arm 2, n = 1.00193004

2 How far etendue is from the textbook

How large is the error on the fringe count? Ten orders of magnitude below the limit. Over a mirror travel of three decades, and through the filled gas cell, the path difference agrees with the textbook to the rounding of double-precision arithmetic.

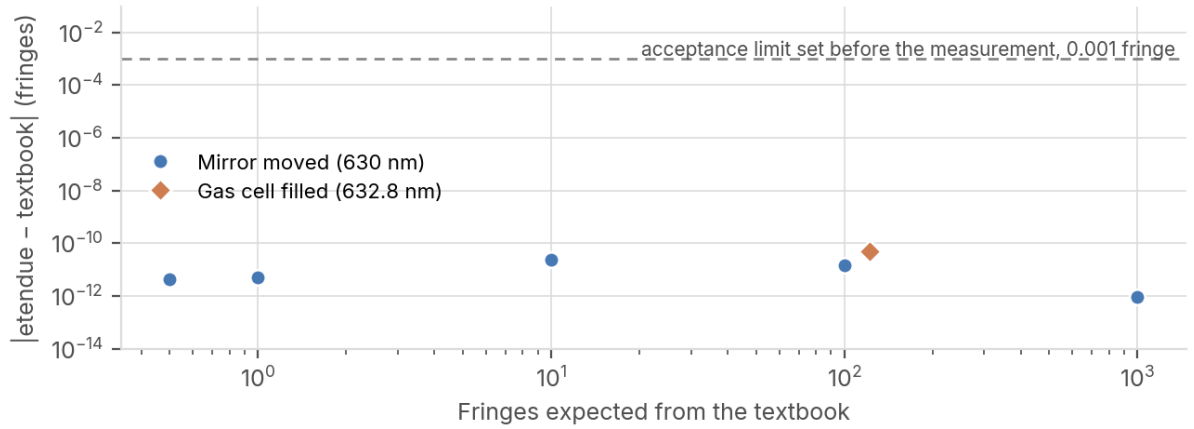


Figure 1. Difference between the fringe count computed by etendue and the textbook value, against the number of fringes expected. Circles: mirror 2 moved by 0.1575 to 315 µm at 630 nm. Diamond: gas cell filled at 632.8 nm. The dashed line is the acceptance limit, 0.001 fringe.

1.5×10^{-8} nm

Largest error on the path difference over six mirror positions, from 0 to 315 µm.

1.000000000

Fringes counted for the 0.315 µm move of the textbook example (1 expected).

122.000000

Fringes counted when the 2 cm gas cell is filled (122 in the textbook).

14 of 16

Comparisons within the limits written before the measurement.

The numbers, and what they do not say

Every comparison, its limit, and the one configuration where etendue declines to answer.

Table 2. Path difference and fringes against the mirror displacement, at 630 nm

Mirror moved (µm)	Textbook (nm)	etendue (nm)	Error (nm)	etendue fringes	Intensity
0	0.0	0.000000	2.3×10^{-9}	0.000000000	1.000
0.1575	315.0	315.000000	2.8×10^{-9}	0.500000000	0.000
0.315	630.0	630.000000	3.3×10^{-9}	1.000000000	1.000
3.15	6,300.0	6,300.000000	1.5×10^{-8}	10.000000000	1.000
31.5	63,000.0	63,000.000000	-9.5×10^{-9}	100.000000000	1.000
315	630,000.0	630,000.000000	5.8×10^{-10}	1000.000000000	1.000

The textbook path difference is twice the mirror displacement, and the textbook fringe count is that path difference divided by 630 nm. The intensity is normalised to a bright fringe: 0 at a quarter-wave move (dark fringe), 1 elsewhere.

Table 3. The gas cell, at 632.8 nm

Quantity	Textbook	etendue	Error
Fringes, cell filled with gas	122	122.000000000	-4.8×10^{-11}
Refractive index, four decimals	1.0019	1.0019	0
Refractive index, unrounded	1.00193004	1.00193004	-6.7×10^{-16}
Index error for 5 fringes miscounted	about 8×10^{-5}	7.910×10^{-5}	—

The index is n = 1 + mλ/(2t), with t = 20 mm. The textbook prints n to four decimals; the unrounded value follows from its 122 fringes.

Table 4. Verdict against the limits written before the measurement

Comparison	Limit	Largest error	Met
Path difference, six mirror positions	≤ 0.63 nm (λ/1000)	1.5×10^{-8} nm	6 of 6
Fringe count, 1 to 1000 fringes	≤ 0.001 fringe	2.4×10^{-11} fringe	4 of 4
Normalised intensity, bright and dark	≤ 10^{-5}	1.9×10^{-22}	2 of 2
Gas cell, fringes and index	≤ 0.001 fringe; same 4 decimals	4.8×10^{-11} fringe	2 of 2
Library project with Fresnel reflections	≤ 0.001 fringe	not read	0 of 2

What this does not cover

The ray tracer gives one path difference for each pair of rays that recombine. It does not draw the circular fringe pattern or its contrast. Those come from the wave model of the laser mode, which is checked against the same textbook in a separate note.

The beam splitter is ideal in the textbook and no power is compared. The gas is modelled, as in the textbook, as a 20 mm column of constant index without walls.

One case that is not met

The interferometer of the project library traces Fresnel reflections at every glass face. 207 distinct beams then reach the detector, not the two of the textbook, and a single path difference is no longer defined. etendue refuses the reading and says why, instead of pairing two stray reflections. In the same trace, the two beams that each visit a single mirror differ by 7.2×10^{-12} fringe at rest and by 1.000000000 fringe after a one-fringe mirror move, read from the traced paths. Traced without Fresnel reflections, the project gives 1.000000000 fringe.

How it was computed

The interferometer is built and traced as the non-sequential editor builds and traces it. The path difference is read by the path-difference function that etendue offers through its API and its assistant. No value was tuned.

References

[1] S. J. Ling, J. Sanny, W. Moebs, *University Physics Volume 3*, OpenStax, Rice University (2016), CC BY 4.0, § 3.6 “The Michelson Interferometer”, Example 3.6.1, the gas-cell example and Exercise 3.6.1. The dark-fringe condition comes from § 3.1 of the same book.