Irradiance of a Lambertian source and ghost reflections of a glass plate

The etendue non-sequential ray tracer compared with exact radiometric view factors and with the Fresnel reflectance at normal incidence

The question

Does etendue put the right amount of light in the right place? An extended source must be sampled so that its irradiance is right at every distance. A ray that meets a glass face must split into reflected and transmitted parts that carry the right power, through any number of reflections.

The reference

Both questions have exact answers. The irradiance of a uniform Lambertian disc on a parallel plane follows from the view factors tabulated by Howell (cases B-12, B-14 and C-41). A plate of index n reflects $R = ((n - 1)/(n + 1))^2$ at each face (Hecht, § 4.6). Summing the reflections in power gives a transmission $2n/(n^2 + 1)$ and ghost-to-image ratios R^2 and R^4.

1 The two systems

The disc emits 1 W with a cosine law. Its irradiance is read on a plane at one to ten source radii, where the inverse-square law is wrong by up to a factor of two. The plate is lit at normal incidence, so polarisation plays no part. Its index goes up to 4 so that the higher ghosts are strong enough to read.

PART	VALUE
Source	disc, 5 mm radius, Lambertian, 1 W, 555 nm
Rays	2 × 2 million per detector distance
Receiver	400 × 400 mm plane at h = 5, 10, 20, 50 mm
Plate	5 mm thick, n = 1.5, 2, 3, 4, no absorption
Beam on the plate	collimated, 2 mm radius, normal incidence

2 How far etendue is from the exact values

How close is the irradiance? Within a third of a percent everywhere, and within the noise of the random sampling. The ghost ratios of the plate agree to fourteen digits, because a reflection that is not too faint is followed deterministically.

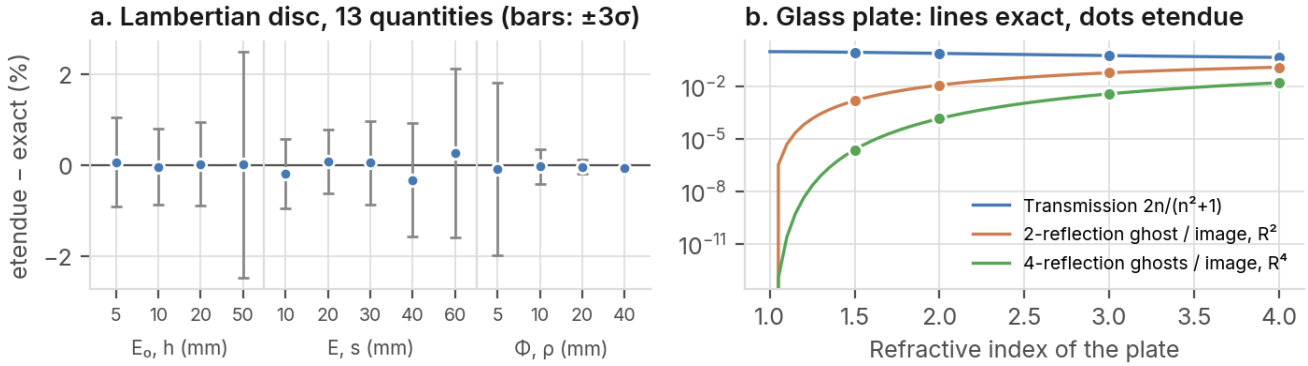


Figure 1. a. Relative difference between etendue and the exact value for the on-axis irradiance E_0 at four distances, the irradiance E(s) off axis at 20 mm, and the power Φ(ρ) within a radius ρ. b. Transmission and ghost-to-image ratios of the plate against its index. Image power for n = 1.5: 0.921600000000 of the source, exact 0.9216.

0.32 %

Largest difference from the exact irradiance or encircled power, over 13 comparisons on a Lambertian disc.

2.6 σ

Largest difference in units of the Monte Carlo noise. All 13 lie within 3σ.

0.001600

Two-reflection ghost of an n = 1.5 plate relative to its image. Exact: 0.0016.

29 of 29

Comparisons within the limits written before the measurement.

References: Howell, Catalog of Configuration Factors; Hecht, Optics, § 4.6 · etendue, November 2026

The numbers, and what they do not say

Every comparison, the noise it was judged against, and what was not met.

Table 2. Lambertian disc, 5 mm radius, 1 W

Quantity	Exact	etendue	Diff. (%)	σ (%)	Diff. / σ	Verdict
On-axis irradiance, h = 5 mm	0.0062337	0.0062377	+0.06	0.33	+0.2	met
On-axis irradiance, h = 10 mm	0.0024619	0.002461	−0.04	0.28	−0.1	met
On-axis irradiance, h = 20 mm	7.2958×10^{-4}	7.2976×10^{-4}	+0.03	0.30	+0.1	met
On-axis irradiance, h = 50 mm	1.2247×10^{-4}	1.2248×10^{-4}	+0.01	0.83	+0.0	met
Irradiance 10 mm off axis, h = 20 mm	4.8812×10^{-4}	4.8718×10^{-4}	−0.19	0.25	−0.8	met
Irradiance 20 mm off axis, h = 20 mm	1.9784×10^{-4}	1.9800×10^{-4}	+0.08	0.23	+0.3	met
Irradiance 30 mm off axis, h = 20 mm	7.7040×10^{-5}	7.7080×10^{-5}	+0.05	0.31	+0.2	met
Irradiance 40 mm off axis, h = 20 mm	3.2407×10^{-5}	3.2303×10^{-5}	−0.32	0.41	−0.8	met
Irradiance 60 mm off axis, h = 20 mm	8.0233×10^{-6}	8.0447×10^{-6}	+0.27	0.62	+0.4	met
Power within 5 mm radius, h = 20 mm	0.055728	0.05568	−0.09	0.63	−0.1	met
Power within 10 mm radius, h = 20 mm	0.19224	0.19218	−0.03	0.13	−0.2	met
Power within 20 mm radius, h = 20 mm	0.49219	0.49199	−0.04	0.05	−0.8	met
Power within 40 mm radius, h = 20 mm	0.79798	0.79746	−0.07	0.03	−2.6	met

Irradiance in W/mm², averaged over the same detector cell as the exact value; power as a fraction of the emitted power. h: distance from the source to the plane. σ: Monte Carlo noise, from two independent draws of two million rays and from the binomial law. A value is met when it lies within 3σ of the exact one.

Table 3. Glass plate, 400 rays: relative difference from the exact value

n	Transmission	Image	2-refl. ghost	4-refl. ghosts
1.5	-2.2×10^{-7}	-8.1×10^{-15}	9.8×10^{-15}	-8.4×10^{-2}
2	5.9×10^{-6}	-6.7×10^{-15}	4.1×10^{-15}	4.1×10^{-2}
3	8.0×10^{-6}	3.4×10^{-15}	0	0
4	3.5×10^{-6}	8.4×10^{-15}	-1.0×10^{-14}	-5.8×10^{-15}

Ghosts are counted relative to the image. In red: not met.

Table 4. Transmission bias, 40 independent draws

n	Mean diff.	Std. error
1.5	1.3×10^{-7}	1.3×10^{-7}
2	-8.7×10^{-8}	6.8×10^{-7}
3	8.1×10^{-7}	9.4×10^{-7}
4	5.5×10^{-7}	1.2×10^{-6}

A disc source of the same radius, 0.01° divergent, 500 rays per draw.

Where the noise is large

The four-reflection ghosts of the n = 1.5 and 2 plates fall below 10^{-4} of the source, where faint reflections are ended at random. They survive in 6 of 400 and 282 of 400 rays, so their value carries a large but known noise, and their difference (−8 %, +4 %) lies within it.

What this does not cover

Oblique incidence, where s and p polarisation reflect differently, and coherent effects such as the Fabry-Perot fringes of a plate. A ray tracer sums powers, as does the reference.

How it was computed

Each scene is built and traced as the non-sequential editor does it, and read by the irradiance map, encircled energy, flux budget and ghost analyses of etendue. No value was tuned. The 100-ray and 400-ray traces of the plate use different random seeds, so that their difference measures the noise of the random ending. Table 4 checks the transmission for a bias over 40 further independent draws.

References

[1] J. R. Howell, *A Catalog of Radiation Heat Transfer Configuration Factors*, 3rd ed., online, cases B-12, B-14, C-41; after H. C. Hottel, *Trans. ASME* 53 (1931) 265-273, and D. C. Hamilton, W. R. Morgan, NACA TN 2836 (1952). [2] E. Hecht, *Optics*, 5th ed., Pearson (2017), § 4.6, reflectance at normal incidence.

Limits set before the measurement: 3σ of the Monte Carlo noise (disc); 3σ of the 100 to 400 ray difference, plus 10^{-9} (plate).