Ray tracing of a 600 lines/mm Littrow spectrograph against the grating equation

Plane reflection grating, 200 mm achromatic doublet used twice, 625 to 680 nm

The question

Does etendue trace a reflection grating right? A spectrograph puts a grating, a camera lens and a detector in a folded path. The grating sends each wavelength in its own direction, and the lens turns that direction into a position on the detector. This note checks both steps against published values.

The references

The grating laws are those of the Diffraction Grating Handbook [1]. They are exact, so the comparison is numerical. The doublet is a catalogue part whose focal length and back focal distance are printed on its public drawing [2]. The instrument is a published amateur spectrograph, with a dispersion given in its user guide [3] and measured by C. Buil [4].

1 The system, as the reference describes it

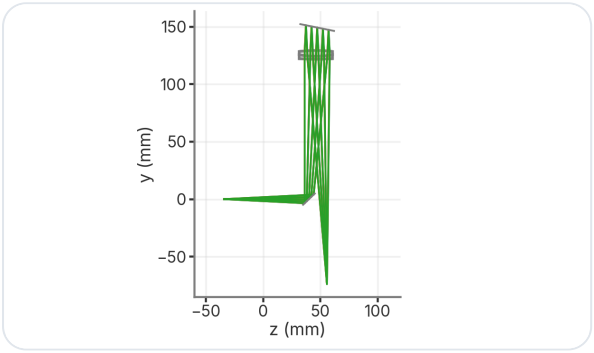
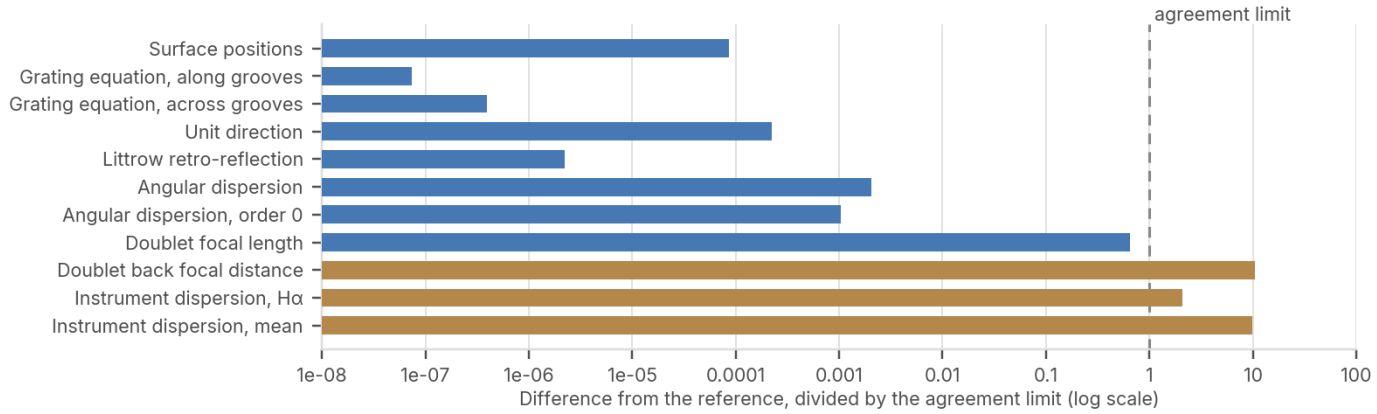


Figure 1. The modelled spectrograph, traced by etendue at 656.3 nm. Light from the slit (left) is folded by a flat mirror, collimated by the doublet, diffracted by the grating (top) and focused back through the same doublet onto the detector (bottom). **Table 1.** Main parameters.

PARAMETER	VALUE
Grating	600 lines/mm, plane, reflection
Camera and collimator	achromatic doublet, 200 mm, 30 mm diameter
Wavelengths	625, 640, 656.3, 670, 680 nm
Aperture, slit	f/10, slit centre and both ends
Angles at 656.3 nm	incidence 8.59°, diffraction 14.15°
Detector pixel	9 µm

2 How close etendue comes 9 of 12 agreement criteria met

etendue meets 9 of the 12 agreement criteria. Every exact law of the grating holds to within rounding of double-precision arithmetic, and the doublet focal length matches its drawing. The three misses are on numbers that the published prescription cannot reproduce, as explained on page 2. **Figure 2.** Each difference divided by its agreement limit; a bar left of the dashed line meets the criterion.



3.9×10^{-16}

Largest error on the grating equation over 9,765 rays, five wavelengths and three orders.

200.003 mm

Focal length of the doublet. The drawing prints 200.00 mm.

0.0730 nm

Dispersion per pixel at Hα. The user guide gives 0.074 nm for a design that works closer to Littrow.

What was compared, and how close

Table 2. Quantities compared with the reference, and the agreement limit written before the measurement

Quantity	Reference	etendue	Difference	Agreement limit	Verdict
Surface positions against the drawing (mm)	0	8.5×10^{-14}	8.5×10^{-14}	1.0×10^{-9}	met
Direction change along the grooves, 9,765 rays	0	7.3×10^{-17}	7.3×10^{-17}	1.0×10^{-9}	met
Direction change across the grooves minus mλ/d, same rays	0	3.9×10^{-16}	3.9×10^{-16}	1.0×10^{-9}	met
Unit length of the diffracted direction	1	$1 \pm 2.2 \times 10^{-16}$	2.2×10^{-16}	1.0×10^{-12}	met
Return on itself at Littrow, λ = 497.7 nm	0	2.2×10^{-15}	2.2×10^{-15}	1.0×10^{-9}	met
Orders that propagate, m = −6 to +6, five wavelengths	65 cases	65 of 65 match	0 mismatch	exact	met
Angular dispersion, orders ±1, relative difference	0	2.0×10^{-9}	2.0×10^{-9}	1.0×10^{-6}	met
Angular dispersion, order 0 (mrad/nm)	0	1.0×10^{-12}	1.0×10^{-12}	1.0×10^{-9}	met
Doublet focal length at 587.6 nm (mm)	200.00	200.0032	+0.0032	±0.005	met
Doublet back focal distance at 587.6 nm (mm)	196.57	196.5174	-0.0526	±0.005	not met
Instrument dispersion at Hα (nm per 9 µm pixel)	0.074	0.0730	-0.0010	±0.0005	not met
Instrument dispersion, 625–680 nm mean (Å per pixel)	0.735	0.7301	-0.0049	±0.0005	not met

Grating laws: the direction of each ray just before and just after the grating, read in the grating frame. Agreement limits are numerical for exact laws and half the last printed digit for published values.

Why the back focal distance misses

etendue gives 196.517 mm, and an independent paraxial matrix calculation with the same indices gives the same value to 10^{-13} mm. Moving every radius and thickness of the drawing by half its last printed digit keeps it between 196.502 and 196.533 mm. The drawing does not say how its 196.57 mm is defined. One reading gives both printed numbers: the focal length at 587.6 nm (200.003 mm) with the back focal distance to the mean paraxial focus of the F, d and C lines, which etendue gives as 196.570 mm. No single wavelength and no real-ray best focus at 587.6 nm does. This match is consistent, not proven; the criterion keeps the single-wavelength definition written before the measurement.

Why the instrument dispersion misses

The real instrument lens is not published, nor the angle between its incoming and outgoing beams (the guide only says the instrument works slightly off axis), so the model replaces the lens with the catalogue doublet and separates the slit from the detector. The model then works 2.80° off the Littrow angle, which lowers the dispersion by 0.77 %. The full difference is accounted for by four factors that etendue measures itself: the angle off Littrow, the doublet focal length at 656.3 nm, the off-axis angle in the camera and the detector position. An independent tracer reproduces the detector positions to 7.8×10^{-14} mm. The miss belongs to the model of the instrument, not to the ray tracing.

Domain of validity

This is geometrical ray tracing of directions and positions. Grating efficiency, order intensities and resolving power are not compared: none is published for this grating.

Table 3. Where the instrument dispersion of the model comes from (reciprocal dispersion at Hα)

Step	Factor	nm/mm
Grating law in exact Littrow, 200 mm camera	—	8.1702
Diffraction angle 14.15° instead of 11.36° (Littrow)	0.98901	
Doublet focal length at 656.3 nm, 200.14 mm	0.99931	
Off-axis angle in the camera, 2.80°	0.99762	
Real ray trace against first order (detector focus, distortion)	1.00643	
Product of the factors, and the value etendue traces	0.99232	8.1075

Reciprocal linear dispersion, Palmer equation (2-17). The first four factors are measured by etendue on the model; the last is what remains between real rays and first-order optics.

References

1. C. Palmer, *Diffraction Grating Handbook*, 8th edition (2020), chapter 2, pp. 21–28, equations (2-1) to (2-17).
2. Public drawing of the 30 mm × 200 mm achromatic doublet, catalogue part 45-415, note 7 (focal length and back focal distance).
3. User guide of the spectrograph, ref. DC0004A, O. Thizy and F. Cochard (December 2006), p. 7.
4. C. Buil, test of the spectrograph with a 600 lines/mm grating, night of 14 November 2006, buil.astrosurf.com/test600/eval_en.htm.