Michelson and Mach-Zehnder interferometers in laser mode, compared with textbook results

He-Ne laser, each arm propagated as a wave: fringe count, gas index, visibility and coherence, fringe period, rings and energy balance

Question

Does etendue compute the fringes of an interferometer correctly when it propagates each arm as a wave? Designers of metrology and alignment benches read the fringe count, the visibility, the coherence length and the fringe period from these maps. This note compares them with the Michelson results of a published textbook and with closed-form results of two-beam interference.

Reference and method

An open textbook gives the Michelson results: one fringe per half wavelength of mirror travel, and a gas index from 122 fringes [1]. Two-beam interference gives the visibility, equal to the degree of coherence, and the fringe period of a tilted mirror [2]. A lossless Mach-Zehnder with equal arms sends all the light to one output [3]. In etendue each arm is propagated as a wave to the detector, where the fields are added. The agreement criteria were set before the measurement.

1 The systems

PARAMETER	VALUE	PARAMETER	VALUE
Source	He-Ne laser, 632.8 nm, 2 mW, line width 0.002 nm	Michelson arms	100 mm, 50/50 beam splitter, plane mirrors
Beam	1 mm radius at 1/e², expanded 5× to 5 mm	Detector	50 mm after the beam splitter
Mach-Zehnder	two 50/50 beam splitters, equal arms	Propagation	each arm as a wave, fields added at the detector

Table 1. The interferometers as entered in etendue in laser mode. For each test the system is modified as a user would do it: a mirror moved or tilted, a gas cell added, the expander removed, a spherical mirror.

2 Deviation of each quantity from its reference

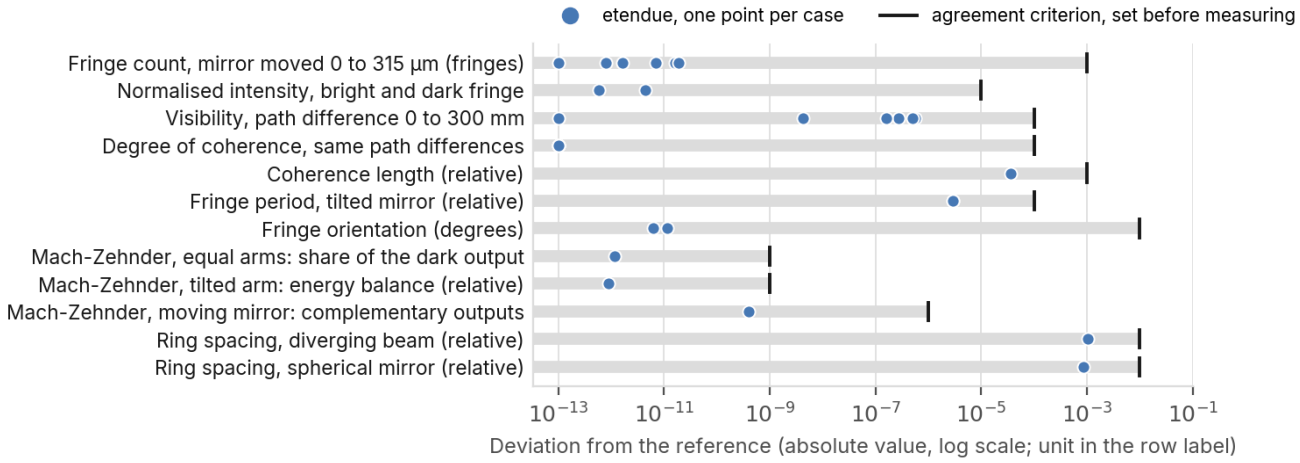


Figure 1. Deviation of etendue from the reference, one point per case, against the agreement criterion of each quantity (black tick). Points at 10^{-13} or below agree to machine precision. The coherence length is 132.996 mm against 133.001 mm for the definition etendue states.

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Quantities within the agreement criterion written before the measurement.

1.9×10^{-11}
Largest error on the fringe count, in fringes, for mirror displacements up to 1000 fringes.

5.7×10^{-7}
Largest error on the fringe visibility for path differences up to 300 mm.

0.5036 mm
Fringe period for a mirror tilted by 0.036°, against 0.5036 mm in theory.

Results against the published references

32 of 32 quantities within their criteria

Table 2. Deviation by quantity, all cases

QUANTITY	VALUES	LARGEST DEVIATION	CRITERION	VERDICT
Fringe count (fringes)	6	1.9×10^{-11}	0.001	met
Intensity, bright and dark	2	4.5×10^{-12}	10^{-5}	met
Visibility	6	5.7×10^{-7}	10^{-4}	met
Degree of coherence	6	$< 10^{-15}$	10^{-4}	met
Coherence length (rel.)	1	3.6×10^{-5}	0.001	met
Fringe period (rel.)	2	2.9×10^{-6}	10^{-4}	met
Fringe orientation (°)	2	1.2×10^{-11}	0.01	met
MZ, dark output share	1	1.2×10^{-12}	10^{-9}	met
MZ, energy balance (rel.)	1	9.0×10^{-13}	10^{-9}	met
MZ, complementary outputs	1	4.0×10^{-10}	10^{-6}	met
Rings, diverging beam (rel.)	1	1.0×10^{-3}	0.01	met
Rings, spherical mirror (rel.)	1	8.6×10^{-4}	0.01	met
Gas cell (fringes; index)	2	3.7×10^{-11}	0.001	met

Largest absolute deviation over the cases, in the unit of the row (relative where stated). Cases: mirror moved by 0, 0.1575, 0.315, 3.15, 31.5 and 315 µm at 630 nm; path differences of 0 to 300 mm; mirror tilted by 0.018° and 0.036° in a parallel 10 mm beam; rings with a diverging beam (mirror moved 30 mm) and with a 20 m spherical mirror. MZ: Mach-Zehnder.

Table 3. Fringe visibility and coherence, 0.002 nm line

Path difference (mm)	$\lvert\gamma\rvert$, reference	Coherence, etendue	Visibility, etendue
0	1.000000	1.000000	1.000000
50	0.800917	0.800917	0.800917
100	0.411482	0.411482	0.411481
133	0.207886	0.207886	0.207885
200	0.028668	0.028668	0.028668
300	0.000338	0.000338	0.000338

$\lvert\gamma\rvert = \exp(-(\pi\Delta\nu\cdot OPD/c)^2/(4\ln 2))$ for a Gaussian line of width $\Delta\nu$. The visibility is slightly below $\lvert\gamma\rvert$ because the two beams no longer have the same size and curvature when one arm is longer.

References. [1] S. J. Ling, J. Sanny, W. Moebs, *University Physics*, Vol. 3, OpenStax (2016), § 3.6 "The Michelson Interferometer": Example 3.6.1 (315 nm per fringe at 630 nm), "Measuring the refractive index of a gas" (2 cm, 122 fringes, n = 1.0019) and Exercise 3.6.1. [2] S. Konijnenberg, A. J. L. Adam, H. P. Urbach, *BSc Optics*, TU Delft, § 5.5 "Temporal coherence and the Michelson interferometer": $I = 2I_0\{1 + \mathrm{Re}[\gamma(\tau)\exp(ik_x x)]\}$ and the Wiener-Khinchine theorem, from which $V = \lvert\gamma\rvert$, $\lvert\gamma\rvert$ of a Gaussian line and the period $\lambda/\sin 2\alpha$ of a mirror tilted by α. [3] Physics LibreTexts, "The Mach-Zehnder Interferometer". Ring spacing of two spherical waves of radii R_1, R_2: $\Delta(r^2) = 2\lambda/\lvert 1/R_1 - 1/R_2\rvert$.

Verdict

All 32 quantities judged meet the criteria written before the measurement. Fringe counts agree to about 10^{-11} fringe and the visibility to better than 10^{-6}, far inside the criteria.

The gas cell gives 122.000000000 fringes for 122 published, and an index of 1.0019. Its faces carry an ideal anti-reflection coating: the reference assumes walls that are the same with the cell empty or full, whereas a bare gas–air face reflects about 9×10^{-7} when full and nothing when empty.

What a user should know

The Michelson as delivered, with its beam expander, gives a fringe period of 1.0063 mm for 1.0071 mm with plane waves. The difference comes from the wavefront that the expander leaves on the beam, which the textbook does not have.

The coherence length depends on its definition. For the same 0.002 nm line it is 133 mm by the definition etendue states (Mandel), 177 mm at half-height of $\lvert\gamma\rvert$ and 200 mm as $\lambda^2/\Delta\lambda$. The visibility at a given path difference does not depend on it.

Limits of validity

With a spherical end mirror, the ring spacing is 0.7 to 0.9 % below the two-spherical-wave result, for convex and concave mirrors of 5 and 20 m. That result assumes a parallel beam, and the beam expander of the Michelson leaves a spherical aberration of about 0.1 wave at the beam radius. Recomputed by exact ray tracing through the expander, outside etendue, the reference for the 20 m mirror differs from etendue by +0.09 %. Without the expander, with a parallel 10 mm beam, the two-wave result holds to 0.02 %.

The references assume ideal, lossless beam splitters and a single-mode source with a Gaussian line. Polarisation effects of the beam splitter are not compared.